Fourier optics

Enno Middelberg
edited and augmented by
Adam Enders

January 13, 2023

This experiment is intended to demonstrate properties of the Fourier transform by means of an experimental setup on an optical bench. Furthermore, properties of the periodic grating used in the experiment are to be determined.

The tasks marked as "Preparation" are to be done at home *prior to* the execution of the experiment (and will be inquired upon if necessary) and the results, as far as possible, should be included in the lab report. Tasks marked as "Exercise" refer to the data obtained during the experiment. Especially the experimentation with the Java applets is very informative, and therefore a necessary part of the preparation. If you do not have the possibility to run the applets, please contact the instructor of the experiment!

Prior knowledge: Geometrical optics, wave optics, Fourier transform, laser, CCD camera, error calculus, experimental skills

Contents

1   Introduction

The importance of the Fourier transform in physics and computer science cannot be overstated. One of its implementations, the Fast Fourier Transform (FFT), is sometimes referred to as the most commonly used algorithm on the planet.

The Fourier transform is an integral transform that is suitable to decompose e.g. a time-dependent signal into its frequency components. The resulting data then can be analyzed with regard to which frequency components are dominant and which are not.

A special case of the Fourier transform is the Fourier series. Here, a periodic function is represented by the sum of sine and cosine functions.

Preparation 1 *Experiment with the "Fourier Series" Java applet[1] (this applet can also be obtained as a zip archive from the instructor). Try to develop a feeling for how the addition and subtraction of individual Fourier components affects the result. Calculate the relative amplitudes of the first 8 coefficients of the Fourier expansion of a sawtooth function:*

$$f(t) = \frac{2h}{\pi} \left[\sin \omega t - \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t \mp \cdots \right] \tag{1}$$

and compose a sawtooth function "by hand" (please provide a screenshot and the coefficients in your lab report).

How does the composition of the Fourier components change when a function is shifted along the x-axis? When does the Fourier series consist exclusively of sine and cosine components?

Generally speaking, the term "frequency" refers to the respective physical quantity known as such. In this experiment, however, no temporal signal is turned into a frequency signal. Instead, a two-dimensional image is decomposed into Fourier components, which are subsequently manipulated and then transformed back again to observe various effects. Therefore, one typically speaks here of "spatial frequencies".

Preparation 2 *Experiment with the "Discrete Fourier Transform Viewer" Java applet[2]. Here you can upload an image (right menu bar, "Load Image"), Fourier-transform it, manipulate the transform (second drop-down menu, "Mouse = Draw [shape]"), and observe the effects on the inverse transformation.*

How does the Fourier transform of a delta component (one white pixel in an otherwise black image) look like? What information (large / small structures) is encoded at different positions in the Fourier plane? How do high- and low-pass filters work? Find common (and I mean common!) examples of the application of the Fourier transform.

2   Experiment Setup

The beam of light from a helium-neon laser (λ=632.8 nm) is widened via a beam expander and then is used to illuminate a cross grating G. The lens L1 in the distance f_{L1} produces an image of the grating in the focal plane. This is the Fourier plane in which the Fourier transform of the grating can be manipulated. After deflection by two plane mirrors, a second lens L2 takes care of the inverse transformation from the Fourier plane. A CCD camera records the resulting image and displays it on a screen. Between the beam expander and the grating is a polarization filter, which can be used to regulate the intensity of the light.
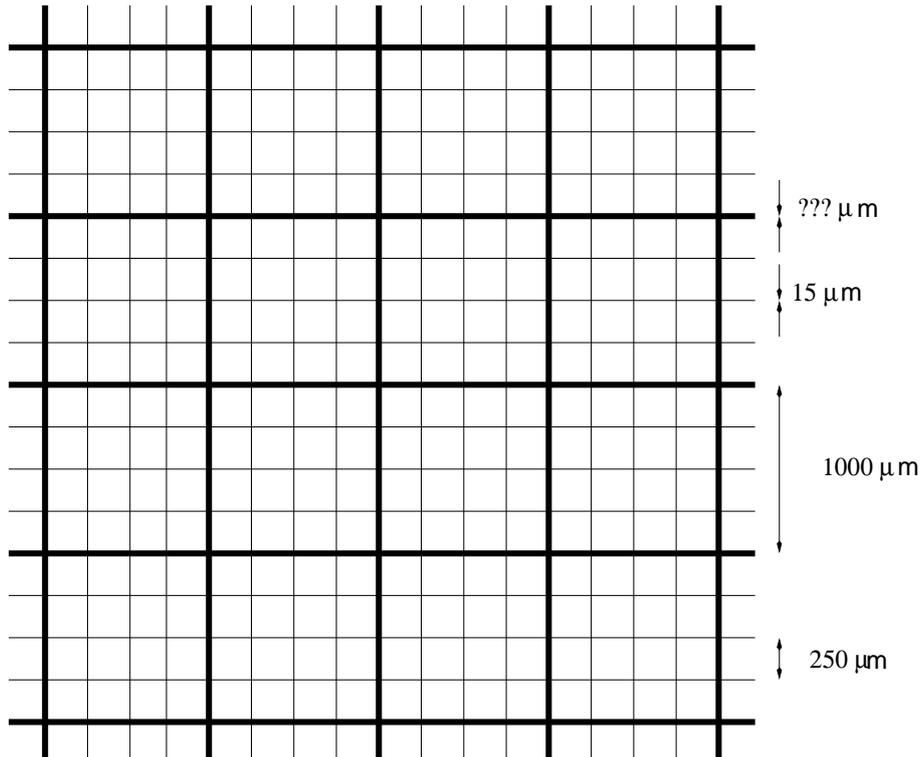
---

[1] http://www.falstad.com/fourier
[2] http://www.falstad.com/fft/

Figure 1: Schematic representation of the grid used in the experiment. The distance between the line centers is $250\,\mu$m, the periodicity is $1000\,\mu$m, the width of the thin lines is $15\,\mu$m and the width of the thick lines is to be determined in the experiment by measurement of the structure function.

The cross grating G consists of a sequence of three thin ($15\,\mu$m) lines and a thicker line, whose exact width is to be determined in the experiment. All line centers have a distance in x- and y-direction of $250\,\mu$m from each other. Thus, the lattice constant is $d = 250\,\mu$m, whereas the periodicity of the lattice is $1000\,\mu$m.

Preparation 3 *Inform yourself about the way a He-Ne laser and a CCD camera works. What is a possible reason if there is no image visible on the screen? Why can the intensity of the laser light be altered using a polarization filter (research whether the light from a He-Ne laser is polarized)? How can you tell if this particular laser is polarized or whether the light is first polarized in the polarizer?*

Unlike during the imaging of a conventional light source, we here can observe effects which are due to the coherence of the laser light. Since two light beams emanating from different points of the lattice have a stable relative phase, the interference pattern resulting from them is stable as well. In the focal plane of L1, this stable interference of the numerous photon beams creates the Fourier transform of the grating, and with the help of a second lens the original image can be restored.

3 Experiment

3.1 Setup

Set up the first part of the experiment as shown in figure 2: laser - polarizing filter - grating - lens L1 - screen. If the distances of the components are chosen appropriately, you will see the
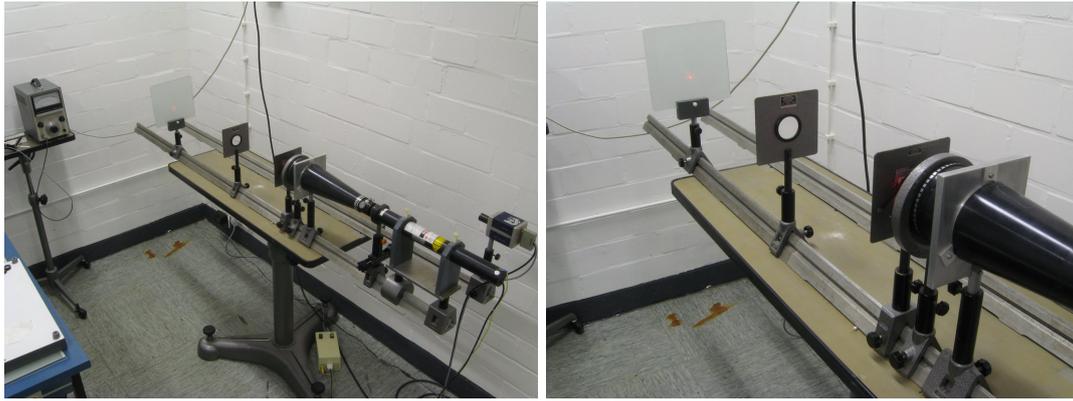
3

Figure 2: *Left:* Overview of the workspace. *right*: First setup. On the screen you should see the Fourier transform of the grid, which is dominated by a cross of points.

Fourier transform of the grating on the screen.

Exercise 1 *Describe the pattern you observe. Then illuminate the grid with the sodium-vapor lamp. To do this, place the lamp directly in front of the polarizing filter and move the grating, lens and screen an appropriate distance to the left. Describe the change in the image and explain your observation.*

Now remove the screen and assemble the other components of the experiment - the plane mirror, the lens L2 and the camera, so that you can see the grating on the screen (Figure 3). If necessary, the intensity of the laser light can be regulated with the polarizing filter. Then place the mask holder into the Fourier plane. The more precise you position the mask holder in x,y and z, the better the results will be in the following part of the experiment.
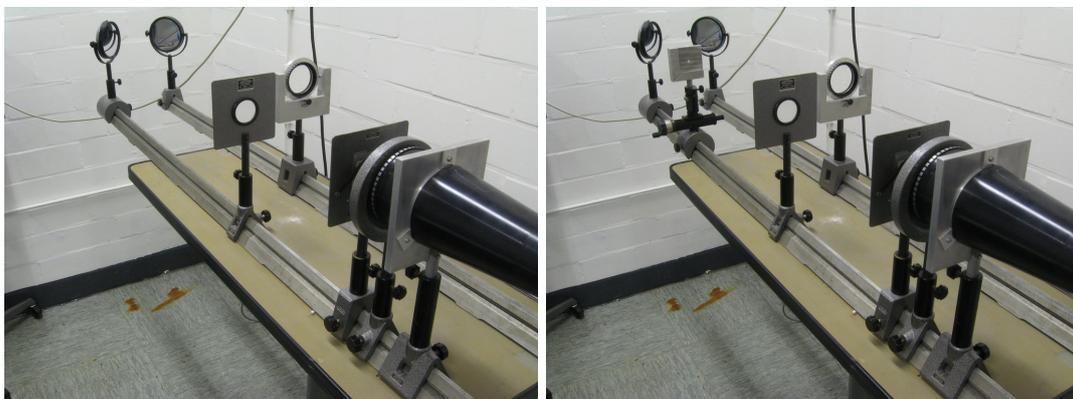

Figure 3: Second setup. *left*: With this setup and suitably dimmed laser you should see an image of the grid on the monitor. *right*: The consequences of the manipulation with masks in the Fourier plane can be observed on the monitor.

3.2 Experiment Part A

Using the masks (Figure 4), you can now manipulate the part of the Fourier transform which is transformed back by L2. Hold a piece of paper directly behind the aperture to verify which orders are still included in the inverse transformation.

1. MA: Block out higher orders. For the interpretation remember your experiments with the 2D Java applet.
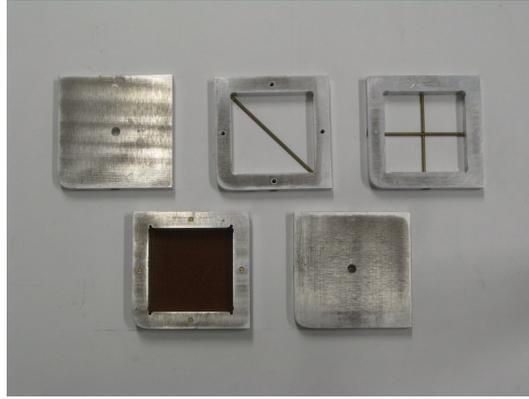
Figure 4: The masks used in the experiment.

2. MB: Block out the zeroth order. Consider which physical quantity is manipulated in the Fourier plane, and which is which one is registered by the CCD camera.

3. MC: Block out the diffraction cross.

4. MD: Block out all orders with $(2m + 1, 2n + 1)$, $m, n = 0, \pm 1, \pm 2, \ldots$. Consider what the Fourier transform of a different (narrower/wider) grid would look like.

5. ME: Block the zeroth and higher orders (here only the maxima with $m, n = \pm 1$ are allowed to pass, which requires skillful adjustment). With the help of paper snippets, additionally block out the remaining maxima (pairwise).

Exercise 2 *Describe exactly what pattern you observe on the screen after inserting each mask. Try to explain how it comes to the observed effect.*

3.3 Experiment Part B

The interference pattern of a grating can be regarded as the multiplication of two functions. The first function represents the interference image of a grating consisting of infinitely narrow lines (=idealized grating), and the second function represents the distribution of the light as it results from the diffraction by a single slit. The intensity distribution of the light in the Fourier plane allows one to draw conclusions about the width of the lines of the grating. By precisely measuring the intensities of the interference maxima in this part of the experiment, the thickness of the broader lines of the grating are to be determined here.

This measurement can only be perfomed beyond the first minimum of the structure function, between the between the 23rd and 31st maximum of the interference image, as here the structure function is fairly strongly dependent on the thickness of the broad lines of the grating. On the other hand, the intensity of the maxima in this area is rather low, and an intensity measurement without prior knowledge of the positions of the maxima will be difficult. Therefore some preparatory steps are are necessary, which will be explained in the following section.

3.3.1 Recording the interference pattern

Place the lens L1 directly in front of the grating, and bring the partially transparent screen into the Fourier plane. Then place the camera in the mounting provided between the screen and the mirror and attach the remote release (Figure 6). The monitor is no longer needed and can be switched off.
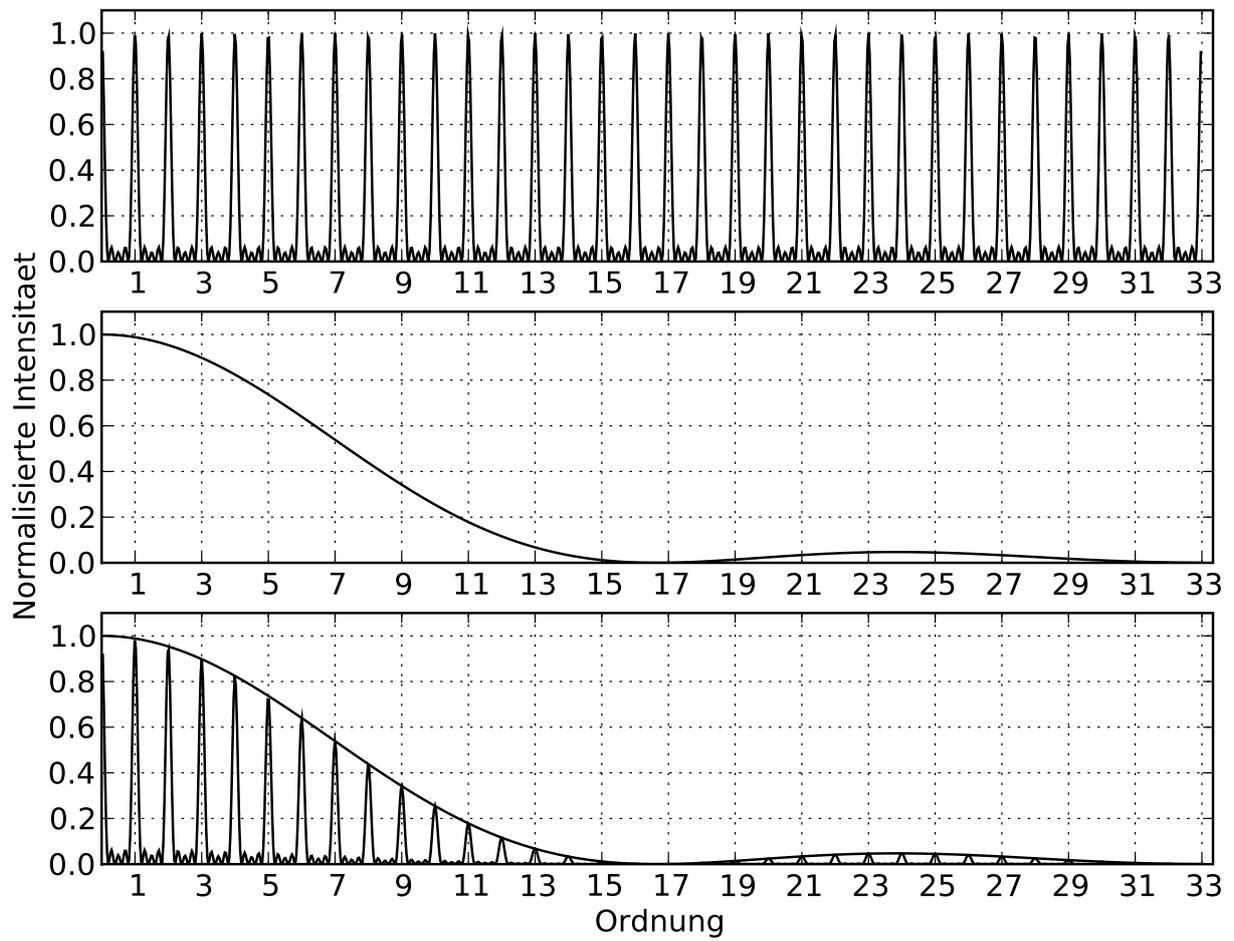
Figure 5: *Top*: Intensity profile of an idealized grid with infinitely thin lines; *middle*: Intensity profile of a single slit; *bottom*: Intensity profile of a real grating. For clarification, the curve from the middle figure is also included here.

Figure 6: Third setup. Note that the lens L1 is directly in front of the grating.

Now choose the maximum zoom level on the camera and select an aperture of f/11. Take a few sample exposures of the screen (with the laser switched off!), varying the camera focus. Repeat this until you have achieved the optimum focus in the area of the screen.

Preparation 4 *Which property is expressed by the f-number* f*? What effects do different values of* f *have on the resulting image?*

With the settings determined experimentally in this way, Part B of the experiment can be carried out. Switch off the room lights, turn on the laser and take a picture of the interference pattern, using the remote trigger. In order to achieve the best possible results during the evaluation, the intensities of the interference maxima of higher order should be as large as possible, but without saturation of the CCD in the vicinity of the lower order maxima. In order to be able to guarantee this, take a series of exposures while varying the exposure time ($t_{\exp} \in [0.05\,\text{s}, ..., 2\,\text{s}]$). From these images you will later determine the most suitable snapshot for your evaluation.

Afterwards, switch off all devices that are still in use (laser, camera, remote trigger) and transfer your data from the SD card of the camera to a storage medium of your choice (a device for reading SD cards is available on site). The part of the experiment taking place in the laboratory is hereby completed.

3.3.2 Image processing and conversion

For image analysis and intensity measurements[3], we will use the program ALADIN[4], a widely used tool among astronomers. As our image still has the wrong file format and also contains superfluous information, some preparations are necessary.

The camera used here saves its RAW (i.e. lossless) images in .dng file format, which we convert for pragmatic reasons with the freeware IRIS[5], popular among amateur astronomers, into a file format suitable for ALADIN. In astronomy, the Flexible Image Transport System (FITS), developed more than four decades ago, still is the golden standard.

To convert a RAW file accordingly, open IRIS and load your image file into the program via "File → Load a RAW file". *Note: Make sure you don't use the regular 'Open' dialog, as this will result in an unwanted FITS file output with multiple layers.*

Preparation 5 *Consider how color images are created in photography and research the purpose of a Bayer matrix in the context of CCDs. What do you expect to see on the RAW image at the position of the interference maxima? Do the observations match your expectations?*

Before we save the file as FITS, we will remove the information previously identified as superfluous from our image. In the case of a monochromatic image, this can be easily achieved via binning of the individual 2×2 Bayer elements. You can find this function in the menu tab "Geometry → Binning". In the dialog box that appears, select a binning factor of two to sum up the pixel intensities of the individual 2×2 areas. Finally, save your image using "File → Save", specifying FITS as the file format in the Save dialog.

Perform the above procedure for each of your recorded images.

---

[3]Strictly speaking, what we measure here is the voltage tapped at the CCD, which corresponds to the number of formed electron-hole pairs, i.e. a dimensionless quantity (typically referred to as *counts*). Since, under the assumption of a constant detector efficiency, this quantity is directly proportional to the intensity, we will adopt the term "intensitiy" in the following sections for reasons of legibility, even though this is formally incorrect.

[4]https://aladin.u-strasbg.fr/

[5]http://www.astrosurf.com/buil/iris-software.html

3.3.3 Image analysis with ALADIN

The FITS files generated in the previous section can now be examined in detail with ALADIN. This chapter serves to explain the basic functions of ALADIN that are relevant here. The numbers and letters in bold refer to the marked areas in the GUI (Figure 7).
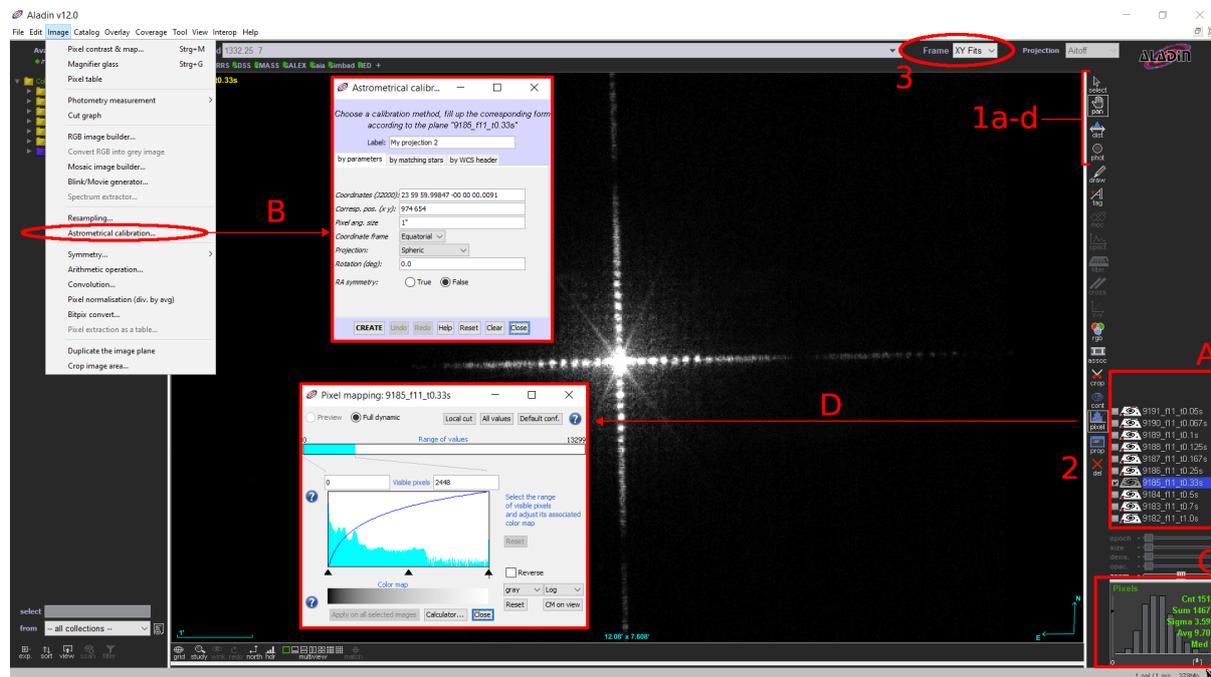

Figure 7: ALADIN's GUI. Areas marked in red refer to the respective references in the text.

Loading files works simply via drag and drop. Drag your file to the image layer area **[A]** at the bottom right of the window. Quite possibly, ALADIN has loaded an astronomical example image on startup. You can safely remove this by selecting the corresponding image layer and clicking on "Del". **[2]**. *Hint:* You can easily have multiple images open at the same time in ALADIN. To do this, simply drag your additional file(s) into the list of image layers. You can display the contents of an image layer by ticking the check mark in front of the corresponding layer. Please note the calibration instructions below!

For the upcoming intensity measurement, ALADIN needs an astrometrically calibrated image. Astrometry obviously does not play a role for our lab recordings, so it is sufficient to provide ALADIN with a "fake" astrometrical calibration. To do this, select the image to be calibrated from the list and select the "Image → Astrometrical calibration" menu item **[B]**. The precise values are irrelevant here, so you can simply leave the default values, and with a click on "Create" in the newly opened window this step is finished. Close the dialog window afterwards.

Important: The astrometric calibration must be performed *as the first step* after opening a new image and *for each image*. If, for example, an intensity measurement is attempted with the phot-tool prior to the calibration, any further measurements will fail and do not provide any meaningful values. The only certain remedy then is to restart ALADIN.

To the left of the image layers you will find a bar with various tools. To us, the first four tools are most relevant, which are briefly explained below:

- **select [1a]**: Allows you to select and move objects you have drawn (such as a distance scale or a photometric aperture, see below), rescale and move them. If you want to select

several such objects at the same time, simply draw a frame around them in the image plane with the Select tool.

- **pan [1b]**: Moves the current image view.

- **dist [1c]**: A "ruler" for measuring distances in the image plane. Since in this experiment we are interested in distances in units of pixels, we still need to select the appropriate scale in Aladin. To do this, select "XY Fits" from the top menu bar next to "Frame". **[3]**.

- **phot [1d]**: With this tool you can define a circular area on the image as a "photometric aperture" and get a statistic about the intensities of the pixels within this area. This is displayed in the lower right area of the window **[C]** and includes number of pixels, sum, standard deviation, average and median of the intensities.

Finally, it should be mentioned that the representation of different intensities is in principle arbitrary and depends on the selected scaling function. ALADIN uses a linear scale as default setting. In order to make particularly faint features more visible on an image, it is useful to increase the range the lower intensity values are mapped to. In ALADIN you can do this with the Pixel tool. For example, you can use the drop-down menu at the lower right edge of the dialog box **[D]**; here the faint maxima will already stand out much more clearly by simply changing to a log or arcsinh scaling.

3.3.4 Data selection

As mentioned above, for the intensity measurements we would ideally use an image with maxima of higher orders as intense as possible, without saturating of the CCD at the positions of the bright maxima of lower orders.

First, we want to get a feel for possible interfering factors for our measurements. For this purpose, the (colored) RAW images are particularly suitable, which you can open with many popular image viewing programs[6]. Take a look at one of the images with moderately long exposure time (e.g. 0.5 s), paying particular attention to the area near the center of the diffraction pattern. Do you notice any phenomena that could significantly influence an intensity measurement? Which maxima are particularly affected here?

Decide on the basis of these considerations, which maximum (preferably of low order) you will use for the following task.

Use the **phot** tool to define an aperture around the maximum you have chosen in a way that you enclose it completely. Be careful not to choose an aperture that has too large an extent, as the spaces between the major maxima are populated with several minor maxima of low intensity (see Fig. 5). *Note:* If you load all images into ALADIN before your measurement, drawing a single aperture is sufficient. By switching to another image in the layer view, the apertures created with the **phot** tool will be automatically updated when you select the appropriate area on the image.

When you determine intensities in this way, your measurement will include not only the laser light, but also additional background effects. Which such effects are known to you or do you notice here? How can you approximately quantify these effects on the basis of your recordings and correct your measurement data? Briefly(!) discuss how it would be possible to improve this correction. For this purpose, also research the terms *flat-field*, *bias-* and *dark frame*.

Exercise 3 *Now plot the (corrected) average intensities as a function of exposure time. In which range does a correlation present itself that corresponds to your expectations? Fit an appropriate*

---

[6]e.g. IrfanView, https://www.irfanview.com/

function in this range. How does the quality of the fit change if you include a longer exposure time?

Since later measurements start with the first maximum, this likewise should not be saturated, if possible. Take a look at the maximum you measured on the saturated and unsaturated images. What differences do you notice?

Now combine your experiences and decide by eye at which exposure time you can perform all measurements of the maxima $n \geq 1$ without being affected by saturation effects. Justify your decision.

3.3.5 Determination of the structure function

With the previous preparations, you now have everything you need to determine the intensities of the maxima on the basis of an image with a suitable exposure time and compare them with the structure functions given here.

Preparation 6 *How do you calculate the standard deviation and standard error? And even more important: what do these two quantities mean? How do they change if more measurements are performed (e.g. 80 instead of 8)?*

For the evaluation, the intensity measurements of the maxima of higher orders will be particularly relevant. Due to the weak intensities of the maxima around n $\approx$ 16, the orders of the maxima (visible with suitable scaling, see above) in the range of interest cannot be determined by "counting". So a prediction must be made for the position of these maxima, which can be accomplished utilizing the clearly detected orders n $= 1 - 9$.

Exercise 4 *Determine the positions of the interference maxima of orders 1-9. Give a realistic estimate of your measurement error.*
Note: *Note that the diffraction pattern in your image will most likely not be perfectly parallel to the CCD-rows of your detector and the higher-order maxima may have a significant offset in the y-direction.*
Then plot the distances of the maxima relative to a suitable reference point on a diagram and derive a functional relationship by linear regression, which expresses the position of a maximum as a function of its order. Can you improve your prediction using the maxima of higher orders?
Also, calculate the mean distance $\bar{x}$ of two adjacent maxima. How can you ensure that the error becomes as small as possible?

Now first measure the intensities of the maxima n $= 1 - 9$. To quantify the measurement error, perform at least eight independent measurements of the fifth maximum, assuming that this error is roughly representative of that of the maxima n $= 1 - 9$. Correct these measurements analogously to 3.3.4 for background effects.

Using the positions derived from the previously determined function, you can also determine the intensities of the weak maxima (n $= 23 - 31$). Provide an estimate of your measurement error analogous to the procedure above, using the 27th maximum.

Exercise 5 *Now plot the intensities in the diagram in Figure 8, normalizing your measured values to the intensity of the first maximum. Note the additional error term due to the normalization!*
Based on the comparison with the given structure functions you can now decide which grid with which line thickness was used in this experiment.
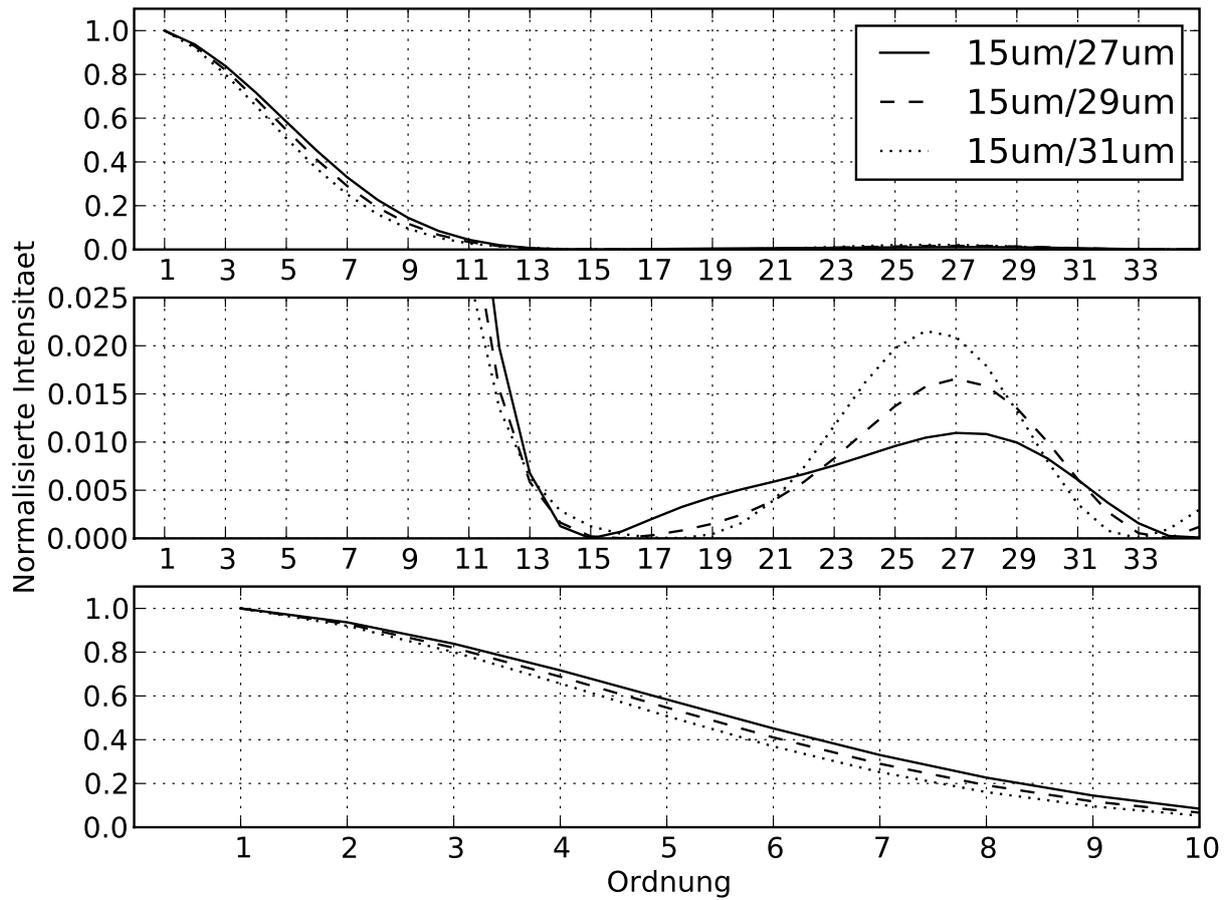
Figure 8: *Top*: Structure function of the grid used in the experiment. grid. *middle and bottom*: Enlarged sections of the top diagram.